CHARACTERISTIC IMPEDANCE OF THE SHIELDED-STRIP TRANSMISSION LINE

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Summary

Simple formulas are given for the characteristic impedance of a transmission line consisting of a conducting strip of rectangular cross section centered between parallel conducting plates at ground potential. The formulas agree to within 1.2 per cent with an exact formula for a zero thickness strip. In the case of finite thickness up to a quarter of the plate spacing, the formulas are expected to be at least that accurate. A family of characteristic impedance curves given in this paper should prove useful to the design engineer.

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I. Introduction

The shielded-strip transmission-line structure shown in Fig. 1 is finding wide-spread use in uhf and microwave circuits. This has been particularly true since the publication of an article by R. M. Barrett explaining the advantages of this configuration, suggesting applications and giving some design data.[1] As explained by Barrett, the shielded-strip line is free of radiation loss and is, therefore, suited for all types of circuits, including high-Q filters. Its form is well adapted to economical methods of manufacture, such as stamping and photo etching.

Despite the large amount of work now in progress with shielded-strip lines, an accurate formula for the characteristic impedance of a line containing a strip of any width and of non-zero thickness has not been available.* An exact formula for a zero-thickness strip has been given,[2] but even a small thickness may affect the characteristic impedance substantially. For wide strips, thickness is taken into account in this paper through the use of an exact fringing-capacitance formula for a semi-infinite plate between parallel ground planes. The write also presents a formula that is accurate for narrow strips of rectangular cross section. This formula and the formula for wide strips permit the computation of accurate characteristic-impedance values for all widths, and for all strip thicknesses up to at least a quarter of the plate spacing. The maximum error is believed to not exceed 1.2 per cent, and for most widths to be considerably less than this.

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[1] See references at end of article.

* The author has been informed that N. Begovich has studied this problem, but his work has not been published. His analysis differs from that of this article in that it involves the derivation of infinite-series expressions for a scalar potential satisfying Laplace's equation in four regions contained within the line. The resulting formula for Z_o is therefore in unclosed form. N. Begovich also used fringing capacitance approach and obtained a formula similar to (1). These results were also given in the unpublished work noted above.

II. Wide-Strip Case

The characteristic impedance of the shielded-strip line may be expressed in terms of the dimensions and the fringing capacitance of a semi-infinite plate as follows[1]

$$Z_o = \frac{10^{12}}{v\left(\frac{0.0885\varepsilon_r\,(2w)}{(b-t)/2} + 4C_f'\right)} \text{ ohms}$$

where w, b and t are dimensions in cm shown in Fig. 1, v is the velocity of wave propagation in cm per second, ε_r is the relative dielectric constant and C_f' is the fringing capacitance in mmf per cm from one corner of the strip to the adjacent ground plane. A more convenient form of this equation is

$$Z_o = \frac{94.15}{\sqrt{\varepsilon_r}\left(\frac{w/b}{1-t/b} + \frac{C_f'}{0.0885\varepsilon_r}\right)} \text{ ohms} \tag{1}$$

The exact formula for C_f' has been found by conformal mapping as follows[3].

$$C_f' = \frac{0.0885\varepsilon_r}{\pi}\left\{\frac{2}{1-t/b}\log_e\left(\frac{1}{1-t/b}+1\right) - \left(\frac{1}{1-t/b}-1\right)\log_e\left(\frac{1}{(1-t/b)^2}-1\right)\right\} \text{ mmf/cm} \tag{2}$$

This equation is plotted in Fig. 2.

As the strip width is decreased, (1) and (2) become inaccurate when the fringing fields on the left and right sides of the rectangular strip interact with each other. Flux plots indicate that excellent accuracy will result for w at least as small as (b-t). However, the discussion in Part IV of this paper shows that (1) and (2) are usable for $w/(b-t) \geq 0.35$, with a maximum error of 1.2 per cent at the lower limit of w.

III. Narrow-Strip Case

Consider a long current-bearing conductor in free space and of arbitrary cross section. At a sufficient distance from the conductor, the magnetic field lines will be circular, irrespective of the cross-sectional shape. Clearly, it is possible to postulate an equivalent circular conductor, as far as inductance per unit length is concerned. This has been done in the case of a rectangular cross section, and the equations relating the equivalent diameter, d_o with the dimensions of the rectangle are given by Flammer[4] and by Marcuvitz.[5] A graph relating d_o to the dimensions of the rectangle is plotted in Fig. 4 from values computed by Flammer. A similar graph is also given by Marcuvitz. Since the same inductance per unit length occurs with a transmission line containing either a rectangular center conductor or a circular conductor of diameter d_o, the characteristic impedance is also the same. However, for Fig. 4 to be valid, it is necessary that d_o be small compared to the spacing between the center conductor and the nearest other conductor.

The following approximate formula is available for the characteristic impedance of a line consisting of a circular conductor of diameter d_o centered between parallel ground planes (Fig. 3a).[6]

$$Z_o = \frac{60}{\sqrt{\varepsilon_r}} \log_e \frac{4b}{\pi d_o} \text{ ohms} \tag{3}$$

This formula becomes exact as d_o approaches zero. However, comparison with a more precise analysis by Wholey and Eldred[7] shows (3) to be within one per cent for d_o as large as b/2.

Hence (3) in conjunction with Fig. 4 provides accurate values for strips of small cross section. For t = 0, d_o is simply w/2, and

$$Z_o = \frac{60}{\sqrt{\varepsilon_r}} \log_e \frac{8b}{\pi w} \text{ ohms} \tag{4}$$

On the basis of flux distributions and the discussion below, (3) is believed to be accurate to within 1.2 per cent for $w/(b-t) \leq 0.35$ and $t/b \leq 0.25$.

IV. Comparison With an Exact Case

The accuracy of (1) and (3) may be tested against the following exact formula valid for t = 0.[2]

$$Z_o = \frac{30\pi K(k)}{K(k')} \tag{5}$$

where K(k) and K(k') are complete elliptic integrals of the first kind, and

$$k = \operatorname{sech} \frac{\pi w}{2b}, \quad k' = \tanh \frac{\pi w}{2b} \tag{6}$$

Figure 5 shows a graphical comparison of (1), (3) and (5). The maximum error occurs at w/b = 0.35 where (1) and (3) intersect, and is only 1.2 per cent. At w/b = 0.20 and 0.50, the error is reduced to 0.4 per cent, while for lesser and greater w/b, the error rapidly approaches zero.

Similar plots of (1) and (3) have been made for strips having t/b up to 0.25, and in all cases, the curves tend to merge together at least as well as in Fig. 5. As one would expect from a consideration of fringing-field interaction, the intersection of the curves remains very near the same value of w/(b-t) = 0.35. A study of flux plots for t = 0 and $t > 0$ leads one to believe that the error at the intersection point will be no greater in the latter case than in the former, and very likely will be smaller. Hence the proper use of (1) and (3) in their assigned parameter ranges is believed to result in an error of no more than 1.2 per cent near w/(b-t) = 0.35, and considerably less at other values of w/(b-t).

V. Graphical Presentation of Z_o

In Fig. 6, a family of Z_o curves are plotted versus w/b with t/b as parameter. The curve for t/b = 0 is exact, the points having been computed from (5).

The other curves are computed from (1) and (3). Equation (1) was used for $w/(b-t) \geq 0.35$ and (3) for $w/(b-t) < 0.35$. It is seen that the effect of thickness on the characteristic impedance is substantial, even for thicknesses only a few per cent of the plate spacing.

VI. Conclusions

Two simple formulas and auxiliary curves are presented for the characteristic impedance of the shielded strip line. By means of these formulas, accuracy sufficient for any engineering purpose is obtainable for all strip widths and for thicknesses up to at least a quarter of the plate spacing. Fig. 6 displays the characteristic impedance in a form that should be particularly useful to the design engineer.

References

1. R. M. Barrett, "Etched Sheets Serve as Microwave Components," Electronics, vol. 25, pp. 114-118; June 1952.
2. F. Oberhettinger and W. Magnus, "Anwendung der Elliptischen Funktionen in Physik und Technik," Springer; 1949.
3. W. R. Smythe, "Static and Dynamic Electricity," McGraw-Hill Book Co., Inc., New York, p. 104, prob. 35; 1939.
4. C. Flammer, "Equivalent Radii of Thin Cylindrical Antennas with Arbitrary Cross Sections," Stanford Res. Inst. Tech. Rep., March 15, 1950.
5. N. Marcuvitz, "Waveguide Handbook," McGraw-Hill Book Co., Inc., New York, pp. 263-265; 1951.
6. S. Frankel, "Characteristic Functions of Transmission Lines," Communications, pp. 32-35; March 1943.
7. W. B. Wholey and W. N. Eldred, "A New Type of Slotted Line Section," Proc. I.R.E., vol. 38, pp. 244-248; March 1950.

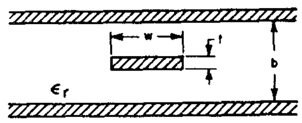

Fig. 1 - Cross section of the shielded-strip transmission line.

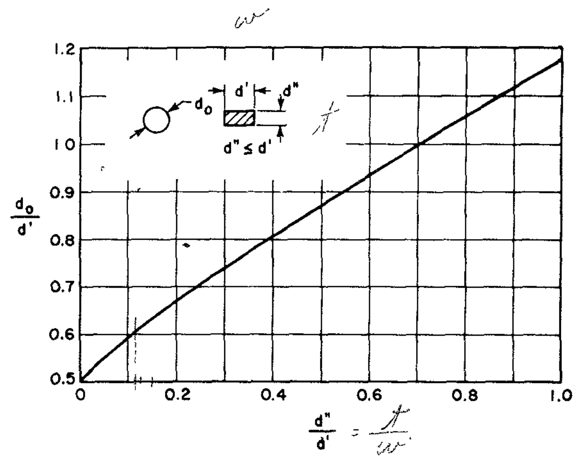

Fig. 4 - Equivalence between a rectangular and circular cross section.

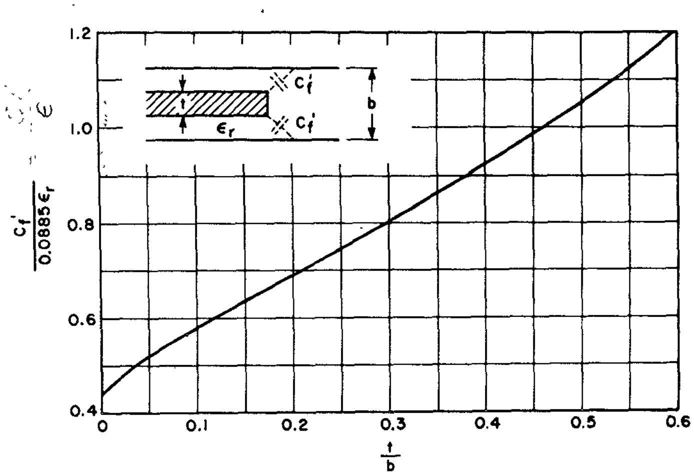

Fig. 2 - Exact fringing capacitance for a semi-infinite plate centered between parallel ground planes. C_f' in mmf/cm.

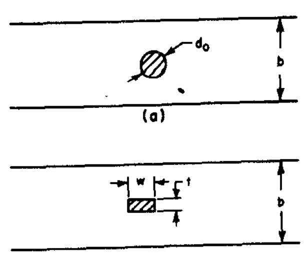

Fig. 3 - Center conductors of small cross section yielding equivalent characteristic impedances.

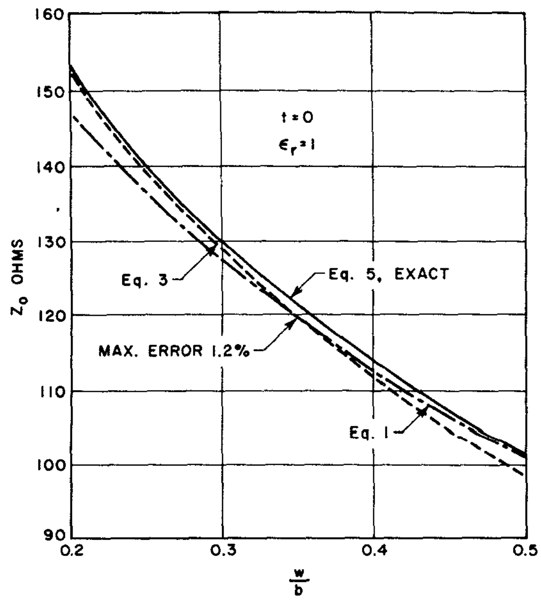

Fig. 5 - Comparison of the two approximate formulas with the exact formula for t = 0.

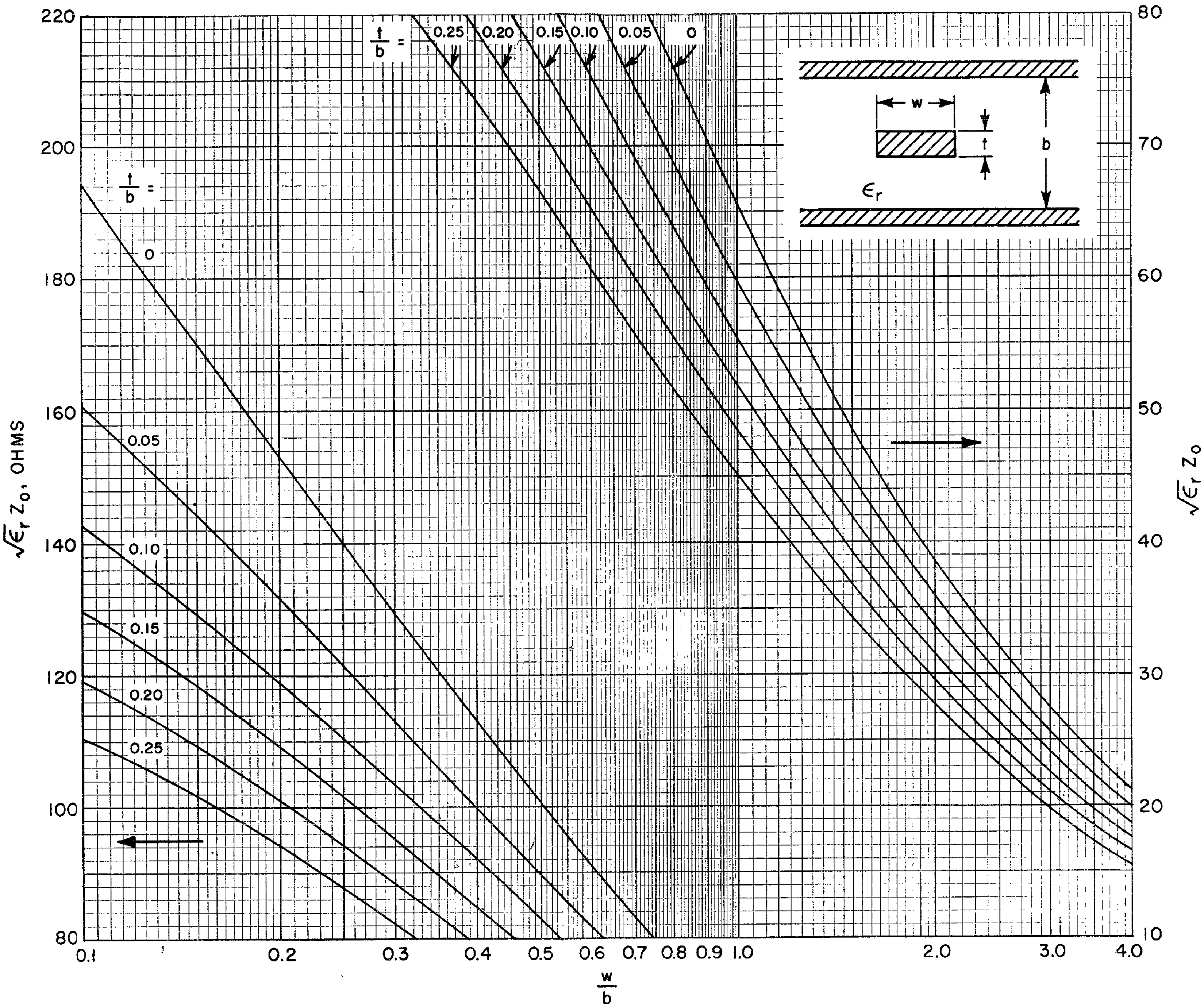

Fig. 6 - Graph of Z_o versus w/b for various values of t/b.